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Effect of Rayleigh number and buoyancy ratio on heat and mass transfer in an inclined parallelogrammic porous enclosure in the presence of magnetic field and heat source

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Abstract— A numerical investigation of the double diffusive natural convection in an inclined porous parallelogrammic enclosure has been performed in this paper. The vertical sloping sidewalls of the enclosure are maintained at different, uniform temperatures and concentrations, while the top and the bottom walls are kept as insulated and impermeable. In addition, the enclosure contains a heat source and a constant magnetic field is applied in the horizontal direction. The governing equations are modeled and are solved using an implicit finite difference method. The numerical algorithm used in the present analysis has been validated and is in good agreement with different benchmark solutions available in the literature. Detailed numerical computations are performed for wide range of Rayleigh numbers, and buoyancy ratio. It is found that the streamlines, isotherms, isoconcentrations and average Nusselt and Sherwood numbers are significantly altered by varying the Rayleigh number and buoyancy ratio.

Keywords: Parallelogrammic enclosure, Rayleigh number, buoyancy ratio, angle of inclination, magnetic field.

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1 Introduction

THE significance of double-diffusive convection through porous materials is primarily stimulated by their sheer existence in many industrial and technological applications such as geophysical systems, electrochemistry and metallurgy. A detailed literature review on thermosolutal convection from the combined buoyancies from temperature and concentration gradients has been reported in the monographs of Nield and Bejan [1]. The investigations on thermosolutal convection in rectangular cavities are abundant in the literature. A combined numerical and theoretical investigation of double-diffusive bifurcation phenomena has been investigated by Mamou et al. [2] in a porous enclosure. Chamkha and Al-Mudhaf [3] conducted a numerical investigation of double-diffusive convection in a tilted porous enclosure containing a heat generating substance. In many engineering applications, the shapes of the enclosure need not be a regularly shaped, such as square or rectangular. Among the non-rectangular enclosures, the parallelogrammic shaped enclosure has several important applications. Natural convection in this type of enclosure is completely different from those taking place in regular shaped enclosures [4, 5].

The influence of various parameters on thermosolutal convection in a parallelogrammic shaped porous cavity has been numerically studied by Costa [6]. More recently, in an inclined parallelogrammic enclosure, Jagadeesha et al. [7] has numerically investigated double diffusive convection to understand the impacts of two tilt angles. In a tilted parallelogrammic porous cavity, the effect of heat generation or absorption and magnetic field on double diffusive convection has not been investigated in the literature. Therefore, the present numerical investigation aims to analyze the effects of Rayleigh number and buoyancy ratio on the thermosolutal convection in a tilted parallelogrammic porous enclosure with an applied magnetic field and heat generation or absorption effects.

2 MATHEMATICAL FORMULATIONS

Consider a parallelogrammic enclosure packed with a fluid saturated porous medium and containing the heat generating or absorbing substance, and is shown in Fig.1 along with the coordinate system. The height and width of the enclosure are taken as H and L respectively, and the enclosure is inclined at an angle α with respect to the x-axis. The inclined left sidewall is maintained at a higher temperature and concentration (T_h & S_h), while the right sidewall is at lower temperature and concentration (T_c & S_c). However, the upper and lower walls are taken to be perfectly insulated and impermeable. The fluid is assumed to obey the Boussinesq approximation and is incompressible. Along the horizontal direction, magnetic field of constant strength is applied.

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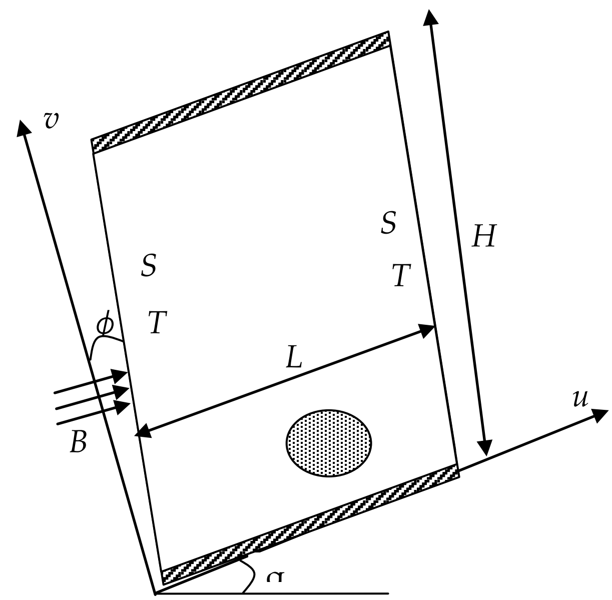

Fig: 1. Physical configuration and coordinate system.

For the computational convenience, the transformation $X = x\text{-}y\tan\phi$, $Y = y$ proposed by Baytas and Pop [4] is used to convert the irregular shaped domain to a regular shaped domain. Applying the Darcy's law, the non-dimensional governing equations are:

$$\frac{\partial^2\psi}{\partial\xi^2}\left(1+Ha^2\cos^2\phi\right)-2\,\frac{\sin\phi}{A}\,\frac{\partial^2\psi}{\partial\xi\partial\eta}+\frac{1}{A^2}\,\frac{\partial^2\psi}{\partial\eta^2}=$$
$$Ra_T\,\cos\phi\left\{\frac{\sin\alpha}{A}\left[\frac{\partial\theta}{\partial\eta}+N\frac{\partial C}{\partial\eta}\right]-\cos(\phi-\alpha)\left[\frac{\partial\theta}{\partial\xi}+N\frac{\partial C}{\partial\xi}\right]\right\} \quad (1)$$

$$\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial\psi}{\partial\xi}\frac{\partial\theta}{\partial\eta}=$$
$$\frac{A}{\cos\phi}\left(\frac{\partial^2\theta}{\partial\xi^2}-2\frac{\sin\phi}{A}\frac{\partial^2\theta}{\partial\xi\partial\eta}+\frac{1}{A^2}\frac{\partial^2\theta}{\partial\eta^2}\right)+G\theta\cos\phi \quad (2)$$

$$\frac{\partial C}{\partial t}+\frac{\partial\psi}{\partial\eta}\frac{\partial C}{\partial\xi}-\frac{\partial\psi}{\partial\xi}\frac{\partial C}{\partial\eta}=$$
$$\frac{A}{\cos\phi}\frac{1}{Le}\left(\frac{\partial^2 C}{\partial\xi^2}-2\frac{\sin\phi}{A}\frac{\partial^2 C}{\partial\xi\partial\eta}+\frac{1}{A^2}\frac{\partial^2 C}{\partial\eta^2}\right) \quad (3)$$

In the above equations, G, A, Ra_T, N and Le are respectively the dimensionless heat generation or absorption parameter, aspect ratio, thermal Rayleigh number, buoyancy ratio and the Lewis number. They are given as:

$$G=\frac{Q_0LH}{\rho Cp\alpha m},\; A=\frac{H}{L},\; Ra_T=\frac{gK\beta_T\Delta TL}{\nu\alpha_m},\; N=\frac{\beta_C\Delta S}{\beta_T\Delta T},\; Le=\frac{\alpha_m}{D}.$$

The relevant initial and boundary conditions are:

$$t=0:\;\; \psi=0,\theta=0;\;\; 0\le\xi\le1,0\le\eta\le1$$
$$t>0:\;\; \psi=0,\;\; \theta=\frac{1}{2},\, S=\frac{1}{2}\; on\; \xi=0$$
$$\psi=0,\;\; \theta=-\frac{1}{2},\, S=-\frac{1}{2}\; on\; \xi=1 \quad (4)$$
$$\psi=0,\;\; \left.\begin{aligned}&\left(\frac{\partial\theta}{\partial\eta}-A\sin\phi\frac{\partial\theta}{\partial\xi}\right)=0\\ &\left(\frac{\partial S}{\partial\eta}-A\sin\phi\frac{\partial S}{\partial\xi}\right)=0\end{aligned}\right\}on\;\; \eta=0,1$$

The problem is to find the functions ψ, θ and S which satisfy the governing equations (1), (2) and (3) and boundary conditions (4). The solution of this problem is depending on the parameters A, Ra, ϕ, α, N and Le.

The overall thermal and solute transport rates at the walls are measured from the following expressions:

$$\overline{Nu}=\int_0^1-\frac{1}{\cos\phi}\left(\frac{\sin\phi}{A}\frac{\partial\theta}{\partial\eta}-\frac{\partial\theta}{\partial\xi}\right)d\eta$$
$$\overline{Sh}=\int_0^1-\frac{1}{\cos\phi}\left(\frac{\sin\phi}{A}\frac{\partial C}{\partial\eta}-\frac{\partial C}{\partial\xi}\right)d\eta \quad (5)$$

Here $\overline{Nu}$ and $\overline{Sh}$ are the global Nusselt and Sherwood numbers respectively.

3 NUMERICAL METHOD AND DISCUSSION OF RESULTS

The model equations (1) – (3) are numerically solved by the finite difference based ADI and SLOR methods. An in-house FORTRAN code is developed to solve finite difference equations of the present model and successful validation has been performed with standard benchmark results.

The effect of variation of Rayleigh number and the buoyancy ratio on double-diffusive natural convection in an inclined parallelogrammic enclosure is investigated numerically in the presence of a magnetic field and internal heat generation/absorption. The cavity aspect ratio (A) is being fixed at 1 during the mathematical formulation. The numerical simulations are displayed through the contours of stream function, temperature and solute concentration, and also through the average Nu and Sh number profiles.

3.1 Streamlines, isotherms and isoconcentration

The influence of Darcy-Rayleigh number on contours of stream function, temperature and concentration is examined and is presented in Fig. 2 for different values of Ra, i.e., Ra = 100 and 1000. The other physical and geometrical parameters are kept fixed at,
$A=1,\phi=30^0,\alpha=30^0, Le=2.0, N=2.0, G=10, Ha=2$. The results clearly shows that, with the combined effect of magnetic

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field and internal heat generation parameter, the flow circulation is slightly amplified with the increase in the Darcy-Rayleigh number and a diagonally elongated larger cell exists in the enclosure for higher value of *Ra*. As the Rayleigh number is increased, isotherms and isoconcentration lines have become widened in the core region of the cavity, which implies that the convection is predominant.

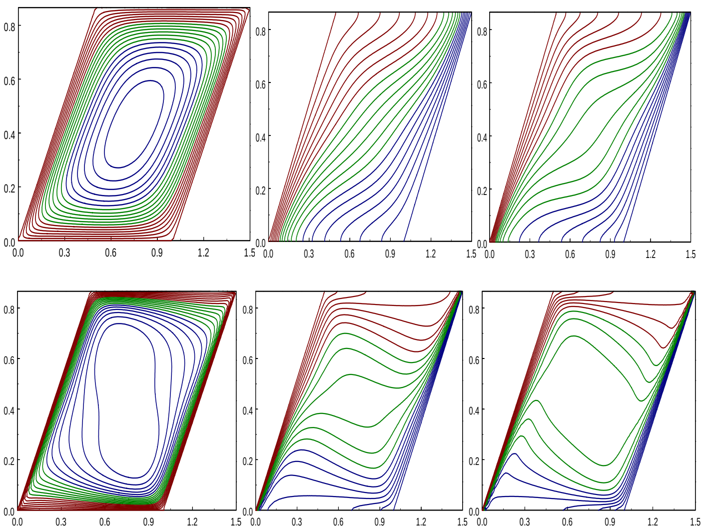


Fig. 2 Streamlines (left), isotherms (middle) and isoconcentration (right) for $A=1, \phi=30^0, \alpha=30^0, Le=2.0, N=2.0, G=10, Ha=2$ at different values of Darcy Rayleigh numbers. (a) $Ra=100, |\psi_{\max}|=5.8$, (b) $Ra=1000, |\psi_{\max}|=20.1$.

Fig. 3 shows the streamlines, isotherms and isoconcentrations for different values of N i.e., -5 and 5. The remaining parameters, namely Ra, φ, α, Le, Ha and G are respectively kept fixed at 500, 30^0, 30^0, 2.0, 1.0 and 10. For N = -5.0, the streamlines in Fig 3 displays a strong diagonal circulating flow pattern with a bi-cellular structure in the middle of the enclosure. Since the solutal buoyancy is dominating for N=-5, the direction flow is clockwise rather than anti-clockwise. The isotherms and isoconcentration also exhibit a similar trend and is consistent with the flow structure. As the buoyancy ratio is increased to a positive value i.e., N = 5.0, a strong flow exists in the anti-clockwise direction due to strong aiding buoyancy forces, which is further evidenced on comparison with the extreme value of the stream function for these two cases of *N*. The streamlines are slightly deformed along the principal diagonal and the isotherms display the standard stratified structure for aiding buoyancy ratio. However, as the magnetic field is applied, the single vortex is divided into two smaller vortices positioned near the adiabatic upper and lower boundaries. The temperature and the concentration lines, in this case, are slightly tilted near the top and bottom walls and are parallel to the side walls in the centre region of the cavity.

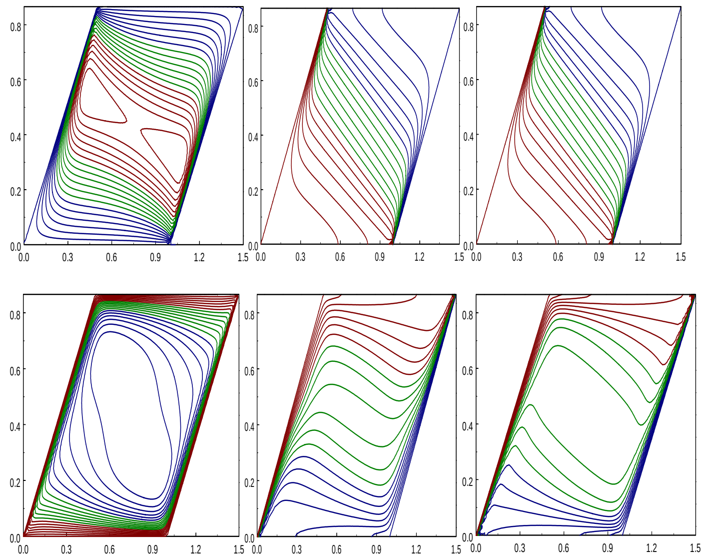
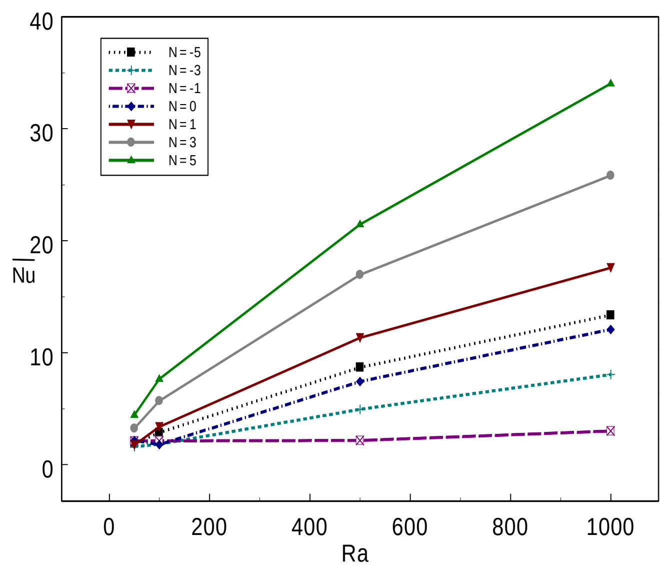


Fig. 3 Streamlines (left), isotherms (middle) and isoconcentration (right) for $A=1, Ra=500, \phi=30^0, \alpha=30^0, Le=2.0, G=10, Ha=1$ at different values of Buoyancy Ratio.(a) $N=-5, |\psi_{\max}|=9.8$ (b) $N=5, |\psi_{\max}|=27.0$.

3.2 Rates of heat and mass transport

The collective effect of Ra_T and N on the total heat and mass transfer rates is depicted in Fig. 4. The remaining parameters of the problem are fixed at a constant value as follows: $\phi=30^0, \alpha=30^0, A=1, Le=2, G=5, Ha=1$. The results are illustrated about the fact that the overall Nusselt and Sherwood numbers receive a marginal rise in accordance with the buoyancy ratio and Rayleigh number. $\overline{Sh}$ record high when compared to $\overline{Nu}$ for each value of the buoyancy ratio. Fig.4 shows that each curve takes a sudden rise at Ra = 100 and attains a maximum $\overline{Nu}$, $\overline{Sh}$ value at Ra = 1000.



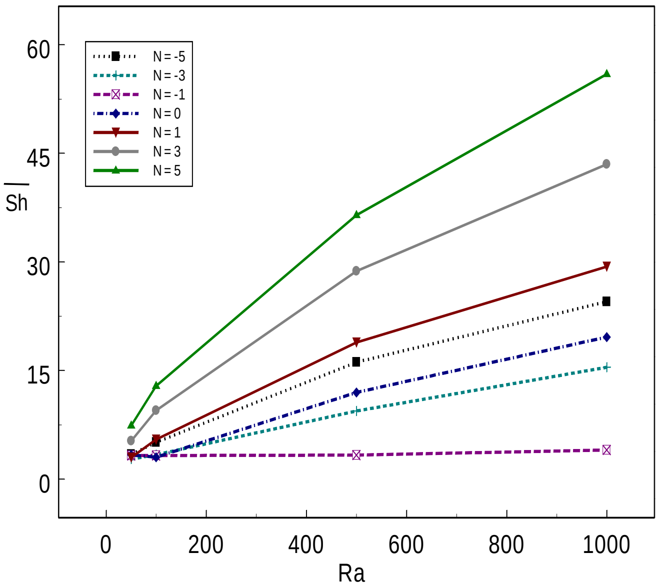


Fig. 4 Effect of Darcy-Rayleigh number and buoyancy ratio on average Nusselt and Sherwood numbers for $\phi = 30^0, \alpha = 30^0, A = 1, Le = 2, G = 5, Ha = 1$.

4 CONCLUSION

The thermosolutal convective flow in a tilted parallelogrammic porous enclosure with a magnetic field and heat generation or absorption effects has been numerically analysed for various Rayleigh numbers and buoyancy ratio. Graphical results for the streamline, temperature and concentration contours for various parametric conditions are presented and discussed. The flow pattern, thermal and solute transport characteristics in the cavity are strongly influenced by the heat generation or absorption effects, cavity inclination angle and the applied magnetic field. As the Rayleigh number is increased, the convection is predominant and is witnessed through the isotherms and isoconcentration lines. The overall thermal and solute transport rates attain maximum value at a critical angle. This critical inclination angle depends on the values of the various parameters involved in the problem. Further, it is observed that the average *Nu* and *Sh* values decreases with heat generation and increases with heat absorption.

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